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Sesión Especial "Teoría de Operadores y Variable Compleja"

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Boundedness and compactness of Hausdorff operators on Fock spaces

Óscar Blasco

Palabras clave: Hausdorff operator, Fock space, summing operator

Mathematics Subject Classification 2020: Primary 30H20; Secondary 47B38, 47B10

Resumen

We obtain a complete characterization of the bounded Hausdorff operators acting on a Fock space F^p_α and taking its values into a larger one F^q_α, $0 < p \leq q \leq \infty$, as well as some necessary or sufficient conditions for a Hausdorff operator to transform a Fock space into a smaller one. Some results are written in the context of mixed norm Fock spaces. Also the compactness of Hausdorff operators on a Fock space is characterized. The compactness result for Hausdorff operators on the Fock space F^∞_α is extended to more general Banach spaces of entire functions with weighted sup norms defined in terms of a radial weight and conditions for the Hausdorff operators to become p-summing are also included.

This is a joint work with Antonio Galbis.

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A sharp estimate for the Hardy number of Koenigs domains

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Palabras clave: Complex Dynamics, Hardy spaces, Koenigs functions

Mathematics Subject Classification 2020: 30D05, 30H10, 30C85

Resumen

For a domain $\Omega \subset \mathbb{C}$, its Hardy number $\mathrm{h}(\Omega)$ is defined as the supremum of all $p > 0$ such that every holomorphic map $f\colon \mathbb{D} \to \Omega$ belongs to the Hardy space H^p (if no such p exists, $\mathrm{h}(\Omega) = 0$). Indeed, the Hardy number of a domain can usually be characterized by its geometric properties. Following this direction, there are several recent contributions by Essén, Hansen, Poggi-Corradi, Kim, Sugawa, Karafyllia, Betsakos, Karamanlis...

The goal of this talk is to analyze the Hardy number of Koenigs domains Ω, that is, those satisfying $\Omega + 1 \subset \Omega$. In particular, using tools coming from potential theory, we will show that for any Koenigs domain Ω, $\mathrm{h}(\Omega) \geq 1/2$ if and only if the logarithmic capacity of $\mathbb{C} \setminus \Omega$ is positive; whereas $\mathrm{h}(\Omega) = 0$ if this capacity is zero.

One of the motivations for this work is that Koenigs domains play a central role in the field of Complex Dynamics. In particular, some implications and further improvements of the latter result can be derived within this area.

This is a joint work with M. D. Contreras, M. Kourou and L. Rodríguez-Piazza.

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Bergman projection on L^p_ν induced by doubling weight

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Palabras clave: Bergman projection, doubling weight, exponential weight

Mathematics Subject Classification 2020: 30H20, 47G10

Resumen

The boundedness of projections on L^p-spaces is a captivating subject that has garnered significant attention over recent decades. Its appeal arises not only from the mathematical difficulties it presents but also from its many applications in pivotal questions within operator theory such us duality relationships or Littlewood-Paley inequalities for weighted Bergman spaces.

In this talk, focusing on the case where $1 < p < \infty$, we delve into characterizing the radial weights ω and ν such that $P_\omega : L^p_\nu \to L^p_\nu$ is bounded, that is

$$\|P_\omega(f)\|_{L^p_\nu} \leq C\|f\|_{L^p_\nu}, \quad 1 < p < \infty. \tag{†}$$

This remains an open problem, even when ν is a standard weight. It is shown that the condition

$$D_p(\omega,\nu) = \sup_{n\in\mathbb{N}\cup\{0\}} \frac{(\nu_{np+1})^{\frac{1}{p}}\,(\sigma_{np'+1})^{\frac{1}{p'}}}{\omega_{2n+1}} < \infty$$

is necessary for (†) to hold, where $\sigma = \omega^{p'}\nu^{-\frac{p'}{p}}$ and $\omega_x = \int_0^1 s^x\omega(s)\,ds$ for all $1 \leq x < \infty$.

Further, we prove $D_p(\omega,\nu) < \infty$ is also sufficient for (†) if the doubling properties $\sup_{0\leq r<1} \frac{\int_r^1 \nu(s)s\,ds}{\int_{\frac{1+r}{2}}^1 \nu(s)s\,ds} < \infty$ and $\sup_{0\leq r<1} \frac{\int_r^1 \nu(s)s\,ds}{\int_r^{1-\frac{1-r}{K}} \nu(s)s\,ds} < \infty$ for some $K > 1$ are satisfied. In addition, we study the one weight inequality $\|P_\omega(f)\|_{D^p_{\nu,k}} \leq C\|f\|_{L^p_\nu}$, where

$$\|f\|^p_{D^p_{\nu,k}} = \sum_{j=0}^{k-1} |f^{(j)}(0)|^p + \int_{\mathbb{D}} |f^{(k)}(z)|^p(1-|z|)^{kp}\nu(z)\,dA(z) < \infty, \quad k \in \mathbb{N}.$$

The inequality (†) is further studied by using the necessary condition $D_p(\omega,\nu) < \infty$ in the case of the exponential type weights $\nu(r) = \exp\left(-\frac{\alpha}{(1-r^l)^\beta}\right)$ and $\omega(r) = \exp\left(-\frac{\widetilde{\alpha}}{(1-r^{\widetilde{l}})^{\widetilde{\beta}}}\right)$, where $0 < \alpha, \widetilde{\alpha}, l, \widetilde{l} < \infty$ and $0 < \beta, \widetilde{\beta} \leq 1$.

This is a joint work with José Ángel Peláez and Jouni Rättyä.

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Dunford property for composition operators on H^p-spaces

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Palabras clave: Composition operators, Local spectral theory, Dunford property

Mathematics Subject Classification 2020: 47A15, 47A11, 47B38

Resumen

A linear bounded operator T acting on a complex Banach space $\mathcal{X}$ has the *Dunford property* if every local spectral subspace of T associated to closed sets are closed. In this talk, we will consider composition operators acting on H^p spaces and will characterize the Dunford property for such composition operators, as well for their adjoints, whenever they are induced by linear fractional transformations of the unit disc. This is a joint work [1] with Eva A. Gallardo-Gutiérrez and Miguel Monsalve-López.

Referencias

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Toeplitz operators and projections

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Palabras clave: Toeplitz operator, Bergman spaces, C*-algebras

Mathematics Subject Classification 2020: 47B35, 30H20, 46L35

Abstract

The C*-algebra generated by Toeplitz operators, acting on the poly-Bergman space of order n, with vertical symbols (and with finite limits at the points 0 and ∞), is isomorphic and isometric to the algebra $\{M(x) \in M_n(\mathbb{C}) \otimes C[0,+\infty] : M(0), M(\infty) \in \mathbb{C}I\}$. Then this algebra looks close to the C^*-algebra generated by n orthogonal projections. This implies that, inside the algebra of all bounded operators acting on the Poly-Bergman space of order n, there are n orthogonal projections that generate a C^*-algebra close related to the C^*-algebra generated by all Toeplitz operators with vertical symbols. In this talk we construct a family of projections with these characteristics. One of them is in terms of Toeplitz operators.

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Título charla

Heisenberg group and Toeplitz operators

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Palabras clave: Siegel domain, Bergman spaces, Toeplitz operators, Heisenberg group

Mathematics Subject Classification 2020: 47B35, 32A36, 22E46, 53D20

Resumen

Bergman spaces and Toeplitz operators acting on them are two very important concrete objects that deal both with operator theory and holomorphic functions. A particularly interesting case is the one given by the $(n+1)$-dimensional Siegel domain: the open subset D_{n+1} consisting of those $z = (z', z_{n+1}) \in \mathbb{C}^{n+1}$ that satisfy $\mathrm{Im}(z_{n+1}) > |z'|^2$. On the other hand, there is a natural biholomorphic action of the Heisenberg group $\mathbb{H}_n$ on D_{n+1}.

In this talk we will consider Toeplitz operators on the weighted Bergman spaces on D_{n+1} with essentially bounded measurable symbols invariant under the action of $\mathbb{H}_n$. Let us denote by $\mathcal{T}^{(\lambda)}\big(L^\infty(D_{n+1})^{\mathbb{H}_n}\big)$ the C^*-algebra generated by such Toeplitz operators, where $\lambda > -1$ is a given weight. We will prove and discuss the following results.

1. The C^*-algebra $\mathcal{T}^{(\lambda)}\big(L^\infty(D_{n+1})^{\mathbb{H}_n}\big)$ is commutative.
2. A simultaneous diagonalizing formula is given for all Toeplitz operators with essentially bounded measurable $\mathbb{H}_n$-invariant symbols.
3. The isomorphism class of $\mathcal{T}^{(\lambda)}\big(L^\infty(D_{n+1})^{\mathbb{H}_n}\big)$ is independent of both the dimension and the weight. Furthermore, we describe such class with a concrete space of functions.

Besides unitary maps given by integral transforms, our results make use of coordinates in D_{n+1} obtained from the so-called moment map of a symplectic action. The latter is computed for the Heisenberg group and its center. This greatly simplify our formulas and proofs, thus highlighting the importance of differential geometric techniques in operator theory. As part of our proof, we obtain a decomposition of weighted Bergman spaces on D_{n+1} as a direct integral of weighted Fock spaces over $\mathbb{C}^n$.

This is joint work with Julio Alberto Barrera-Reyes, Centro de Investigación en Matemáticas.

Referencias

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Recovering bicomplex holomorphic funtions

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Palabras clave: Bicomplex holomorphic functions, harmonic functions

Mathematics Subject Classification 2020: 30G35, 31B05, 32A30

Resumen

In [3], Shaw writes a proof of a classical statement of Ahlfors ([1],p. 27-28) concerning with the reconstruction of a holomorphic function from its real or imaginary parts (without Cauchy-Riemann equations). In the same paper, Shaw defines a complex harmonic function, that is, a smooth function $U : \Omega \subset \mathbb{C}^2 \to \mathbb{C}$, in the domain Ω that satisfies the complex Laplace equation

$$\frac{\partial^2 U}{\partial z_1^2} + \frac{\partial^2 U}{\partial z_2^2} = 0.$$

In this talk we give another proof of the statement of Ahlfors and we show that the bicomplex holomorphic theory is well suited for complex harmonic functions. Moreover, we recover V, a complex conjugate harmonic function of U, such that $U+\mathbf{j}V$ is a bicomplex holomorphic function, see [2], (V is recovered without complex Cauchy-Riemann equations).

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Spectral Asymptotics for the Dirichlet-to-Neumann Operator on the Sphere

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Palabras clave: Dirichlet-to-Neumann operator, Szegő limit theorems

Mathematics Subject Classification 2020: 58C40, 58J40

Resumen

The spectrum of the Dirichlet-to-Neumann operator on the sphere (associated with a potential in the ball) consists of clusters of eigenvalues around the sequence of natural numbers. By results of Weinstein and Guillemin [1, 4], the distribution of eigenvalues in the clusters can be described asymptotically by a sequence of distributions β_ℓ on the real line, the so-called band invariants. In this work we compute the first three band invariants, using a symbol calculus developed in [3]. The expressions involve the Radon transform of differential expressions of q restricted to the sphere. A key contribution is the computation of the expansion of the Berezin symbol of the operator, associated to coherent states on the sphere. This is joint work with Salvador Pérez Esteva and Carlos Villegas Blas (IMATE-UNAM)

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